Specification, Design and Verification

Kais Klai and Walid Gaaloul

Objectives

1. Object Oriented Design
   - To describe the activities in the object-oriented design process
   - To introduce various UML models that can be used to describe an object-oriented design
   - To show how to use OCL to guarantee the models' constraints

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2. Formal Modeling and Verification
   - How to model a concurrent system (using Petri nets)
   - How to express behavioral properties (LTL)
   - How to check a property on a system

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   - How to model a concurrent system (using Petri nets)
   - How to express behavioral properties (LTL)
   - How to check a property on a system
3. Test
   - Test of Object Oriented applications
   - Unit, Integration and Validation Test

Organisation

- 14h lecture (CM)
- 10h30 Tutorials (TP)
- 10h30 Tutorials (Project)
- Evaluation :
    - 1 exam (DE) (66.66%)
    - a project (33.33%)

Formal Specification and Verification of Concurrent Systems

Kais Klai

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Outline

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System
Property

Some Properties

- Reachability: A certain situation can be reached
  x may be zero, each instruction can be executed
- Invariant: Each state respects some good property
  x is never equal to zero, an array never overflows
- Safety: Something bad can never happen
  I access the file if I enter the correct PIN
- Liveness: Something good can always happen
  the program terminate, the message will eventually arrive to the destination, the program always returns to the initial state
- Fairness: Something good happens infinitely often
  If a process asks to enter to a critical section infinitely often, it will access it infinitely often
- ...

Context

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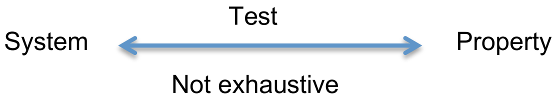

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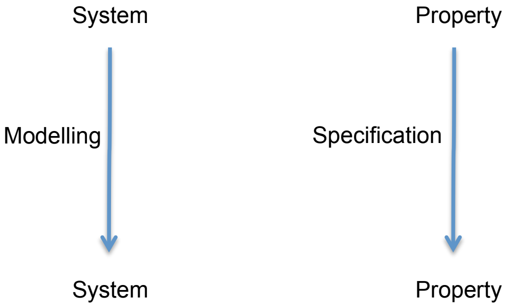
System
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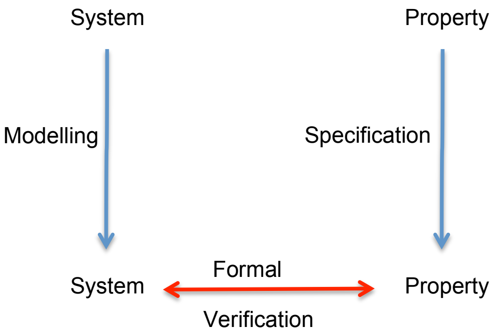
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Formal Verification

1. Theorem Proving
   - Logical description of the system
   - Prove properties by deduction
   - Not fully automatic

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1. Theorem Proving
    - Logical description of the system
    - Prove properties by deduction
    - Not fully automatic
2. Model Checking
    - Exhaustive verification
    - Fully automatic
    - Counter-examples

Example: Mutual Exclusion Algorithm

Global variables: req_P and req_Q

Process P

1. $req_P \leftarrow 1$
2. wait($req_Q = 0$)
3. Critical Section
4. $req_P \leftarrow 0$

Process Q

1. $req_Q \leftarrow 1$
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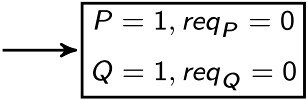
Properties to be checked:

1. Mutual exclusion
2. Fairness
3. Order

Example: Reachability State Space



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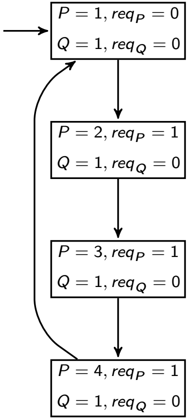

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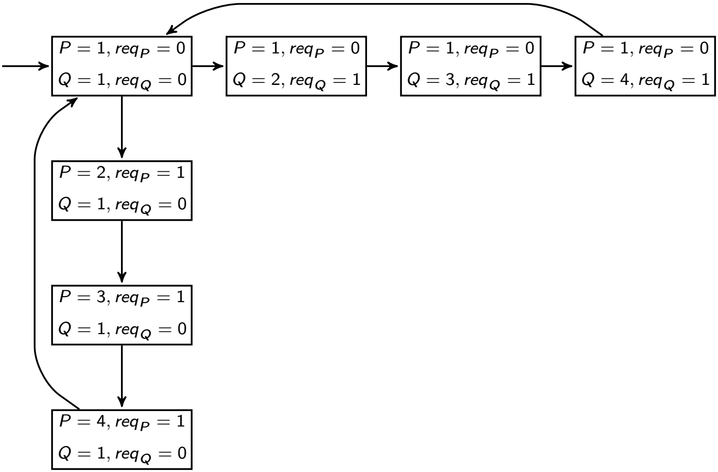

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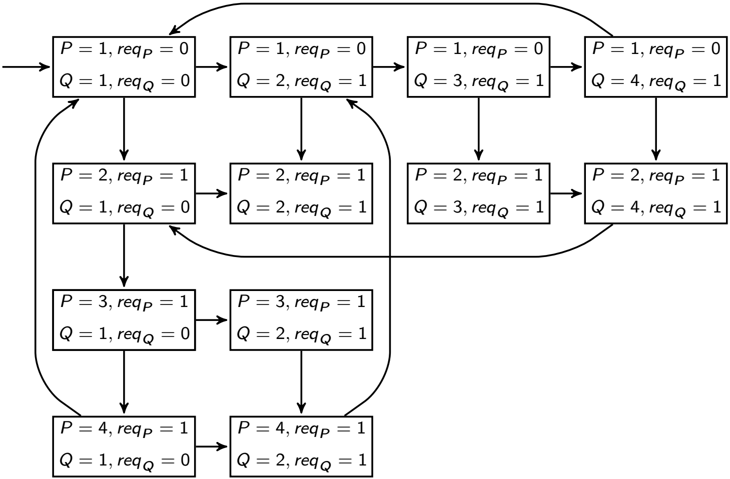

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To check this property we browse the set of reachable states. We need reachable states only, not the transitions between states.

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Outline

Model checking of finite state systems

Principle

1. Design the system with a model $\mathcal{M}$ and design a property φ
2. $\mathcal{M} \models \varphi$? if no, a counter-example σ
3. Analyse the result:
    - If yes, **OK**
    - If no, refine $\mathcal{M}$ using σ and go to (1).

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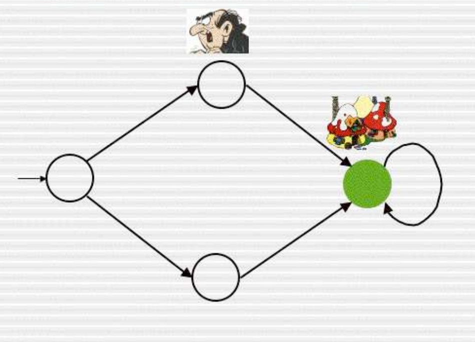
Approach

- State space traversal (Labeled Transition System)

Example

Gargamel !!!

Is there any safe path?



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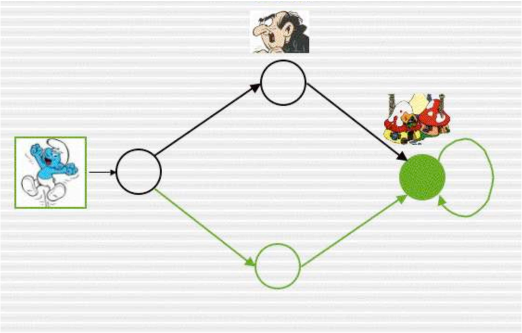
YES



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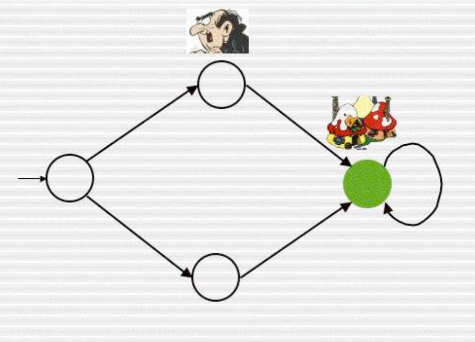
Are all the paths safe?



Example

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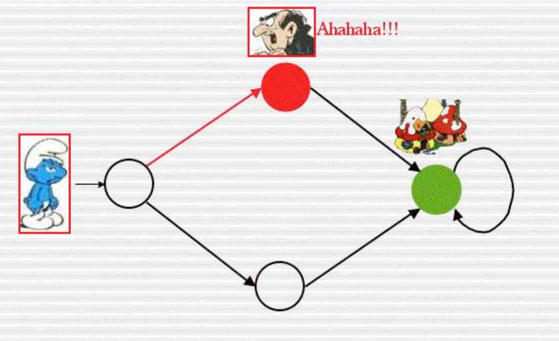
NO



Formal Specifications

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Advantages:

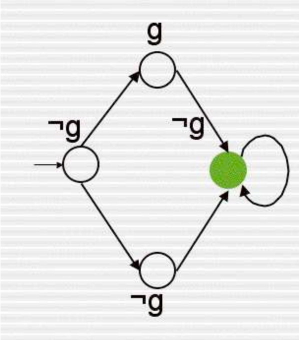
- unambiguous
- generic
- allows for automatic verification

Example

Let's be serious 5 minutes

Example

- *A*: for all paths
- *E*: there exists a path
- *G*: always
- *g*: *Gargamel*
- $\neg$: negation



The formula $EG\ \neg g$ is satisfied by the model

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Ingredients

- $\mathcal{M} =$ The behavior of the System
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Drawbacks

- Finite LTSs
- Requires formal expertise
- State space explosion problem

State space explosion problem

Reduction Techniques

- On-the-fly construction
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    - $a\bar{b}.a\bar{b}.a\bar{b}.ab.ab.ab\ldots$
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S=(C,V,A,T)

- C: Control States
- V: Variables
- A: Actions on V
- T: Transitions

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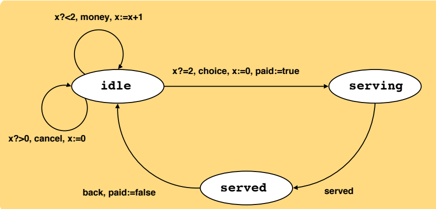

Labeled Transition System (LTS)

LTS = Semantics of the system

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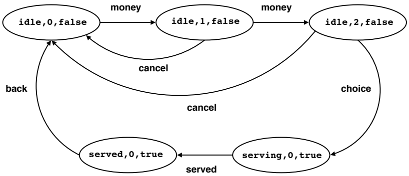

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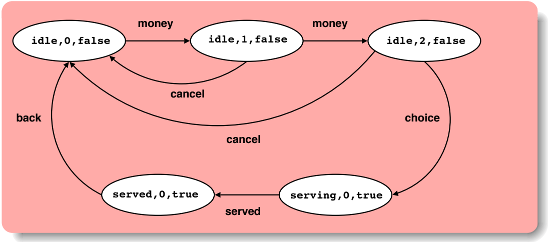
Executions of the system

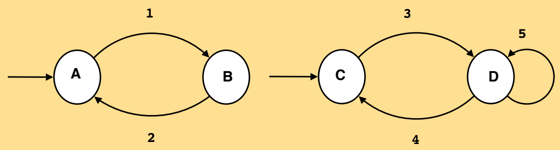


- $(i, 0, f) \xrightarrow{money} (i, 1, f) \xrightarrow{money} (i, 2, f) \xrightarrow{choice} (sg, 0, t) \ldots$
- money, money, choice, served, back, money

$L(S)$ = Language of S= The set of executions of S

Concurrency

Asynchronous product



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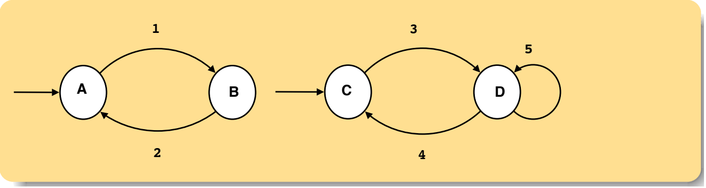
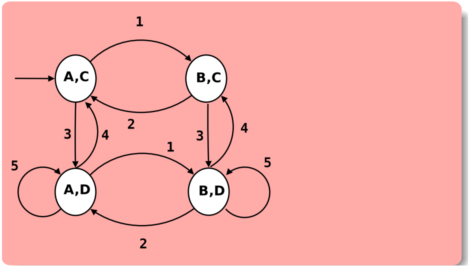
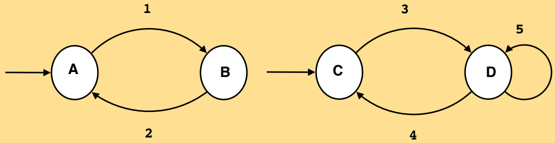




Concurrency

Synchronous product



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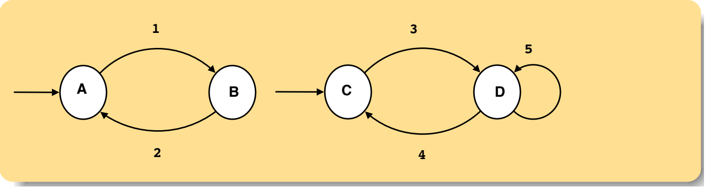
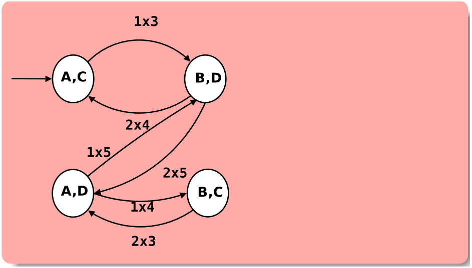
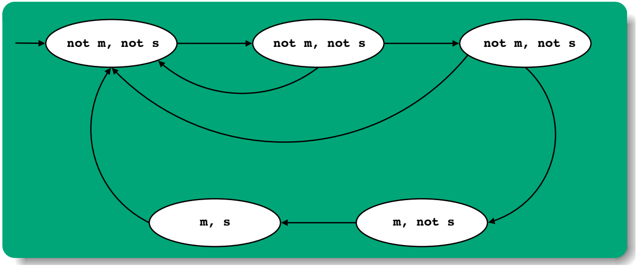




Kripke structure



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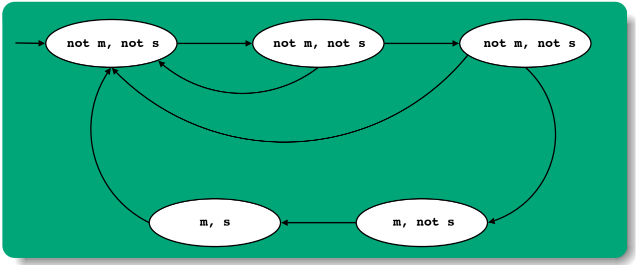
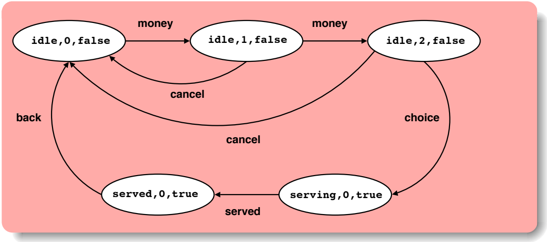




Exercice: The Lift Example

The lift controller system (for 3 floors) is defined by:

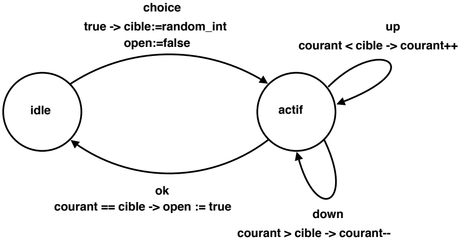
1. the controller saves in memory the current and the target floors.
2. in active mode, when the target floor is reached, the doors are opened and the controller switches to the idle mode.
3. in active, when the target floor is greater than the current one, the controller raises the lift.
4. in active, when the target floor is lower than the current one, the controller lowers the lift.
5. in the idle mode, it may be that someone enters the lift and choose a new target floor. The elevator then closes the doors and becomes active.
6. initially, the elevator is at floor 0 and in the idle mode.

Questions

1. Design the system using a state machine (formal definition and the figure).
2. Define and draw the corresponding transition system.

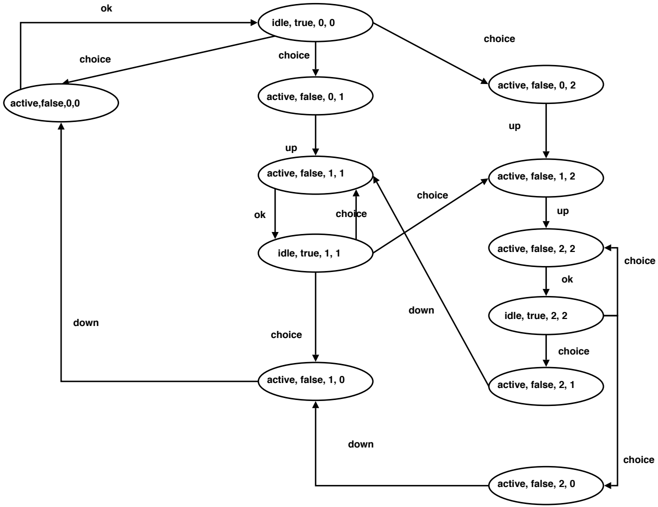
State Machine

- V= `courant: int[0...2]`, cible: int[0...2], open: bool
- `random.in` $\in$ `[0...2]`



The Lift Example

Labeled Transition System



Outline

Plan

4. Formal Specifications
   - Petri nets
   - Coverability Graph
   - Linear Temporal Logic (LTL)

Petri Nets [Petri 73]

Syntax

Definition

A Petri net is 5-tuple $N = \langle P, T, F, W, m_0 \rangle$ where:

- P is a finite set of places (cercles) and T a finite set of transitions (squares) with $(P \cup T) \neq \emptyset$ and $P \cap T = \emptyset$,
- A flow relation $F \subseteq (P \times T) \cup (T \times P)$,
- $W : F \rightarrow \mathbb{N}^+$ assigns a weight (> 0)to any arc.
- An initial marking m_0 where a marking m is a mapping $m : P \rightarrow \mathbb{N}$.

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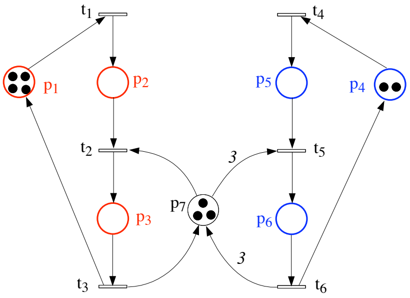
Incidence matrix C: $\forall (p, t) \in P \times T : C(p, t) = W(t, p) - W(p, t)$
Notation: $C(p, t) = Post(t, p) - Pre(t, p))$

Petri Nets: an example



$$
\begin{bmatrix}
-1 & 0 & 1 & 0 & 0 & 0 \\
1 & -1 & 0 & 0 & 0 & 0 \\
0 & 1 & -1 & 0 & 0 & 0 \\
0 & 0 & 0 & -1 & 0 & 1 \\
0 & 0 & 0 & 1 & -1 & 0 \\
0 & 0 & 0 & 0 & 1 & -1 \\
0 & -1 & 1 & 0 & -3 & 3
\end{bmatrix}
$$

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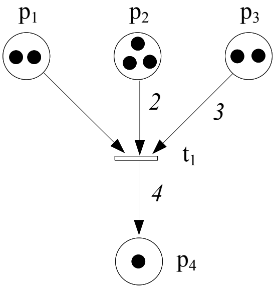

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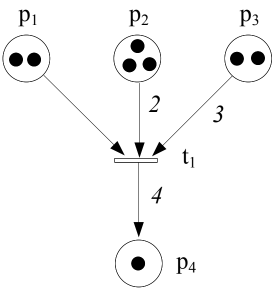

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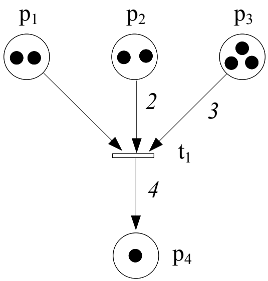

firable

Petri Nets: Semantics

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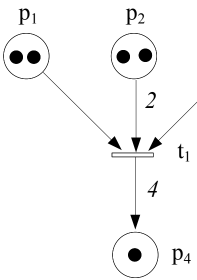

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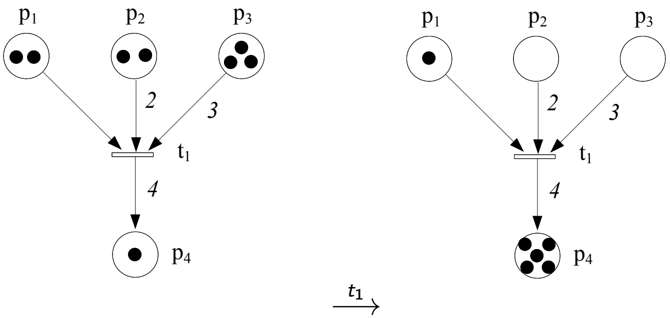
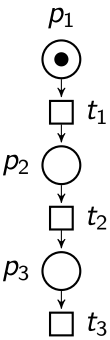


Petri Nets: Expression Power

- Causality



Petri Nets: Expression Power

- Conflict/Choice

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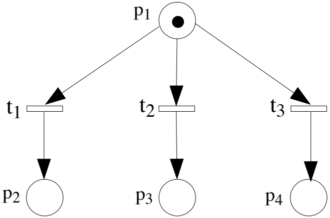

Petri Nets: Expression Power

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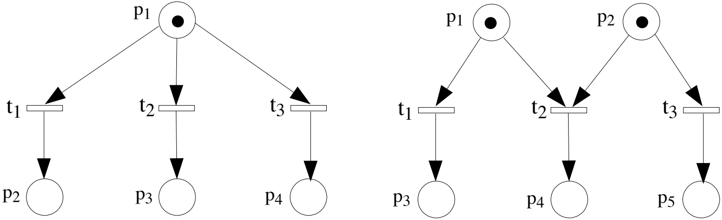

Petri Nets: Expression Power

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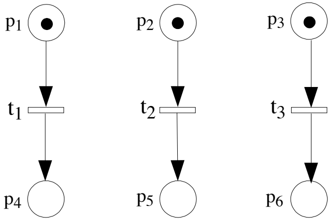

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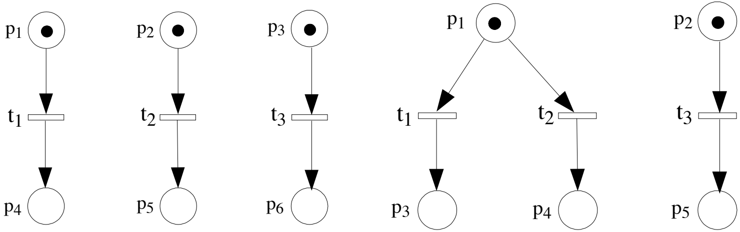
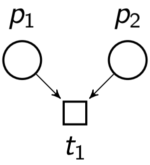
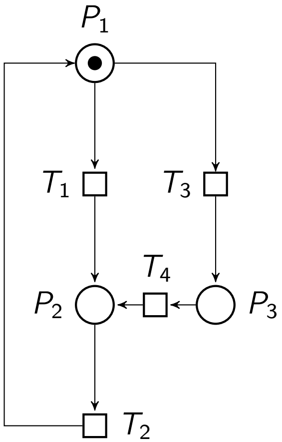


Petri Nets: Expression Power

- Synchronization

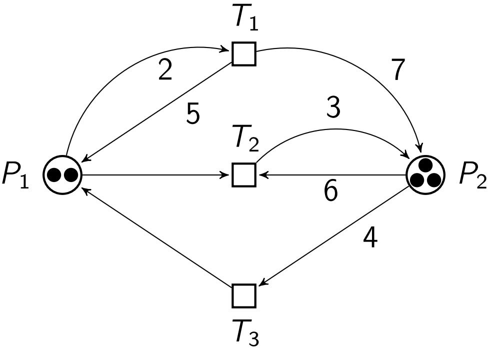


Petri Nets: exercice 1



1. Give the Pre, Post and the incidence matrices of this Petri net.
2. Which are the fireable transitions from the initial marking?

Petri Nets: exercice 2



1. Is T_1 fireable from the initial marking? If yes, which is the reachable marking?
2. Give the incidence matrix of this Petri net.
3. Check formally the fireability of the transition T_1. If T_1 is fireable, then compute the reachable marking formally.

Petri Nets: Semantics (Cont.)

- $\sigma = t_1 \ldots t_n \in T^*$ is fireable at m_0 (denoted by $m_0 \xrightarrow{\sigma}$ iff $\exists m_1 \ldots m_n$ s.t. $m_0 \xrightarrow{t_1} m_1 \longrightarrow \ldots \xrightarrow{t_n} m_n$

Petri Nets: Semantics (Cont.)

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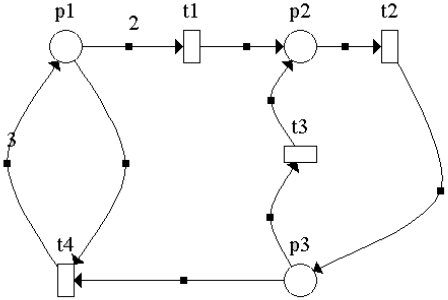
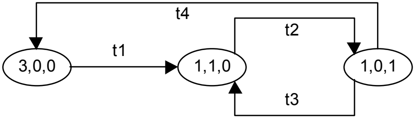
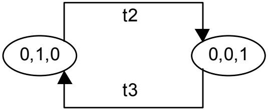
- $L(N, m_0) = \{\sigma \in T^* \mid m_0 \xrightarrow{\sigma}\}$
- $R(N, m)$ = the set markings reachable from a marking m of N
- the reachability graph is a LTS $\langle S, A, \rightarrow, s_0 \rangle$ s.t.
    - $S = R(N, m_0)$
    - $A = T$
    - $s_0 = m_0$
    - $(s_1, t, s_2) \in \rightarrow$ iff $s_1 \xrightarrow{t} s_2$



initial marking $(3, 0, 0)$, then $(0, 1, 0)$

Petri Nets: Reachability Graph

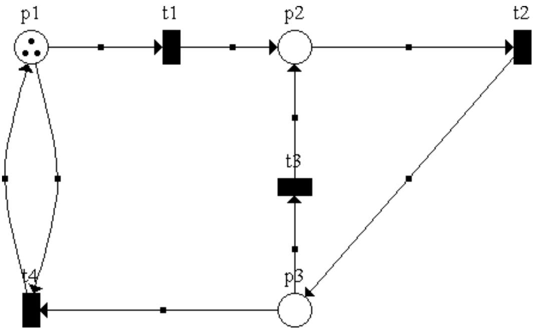




$m_0 = (3, 0, 0)$

$m_0 = (0, 1, 0)$

Petri Nets: Reachability Graph



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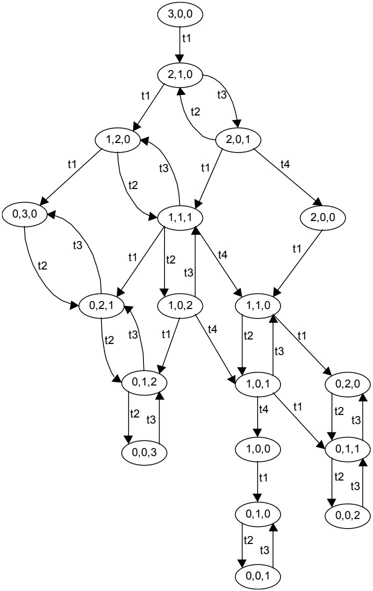

Example: Mutual Exclusion Algorithm

Global variables: req_P and req_Q

Process P

1. $req_P \leftarrow 1$
2. `wait(`$req_Q = 0$`)`
3. `Critical Section`
4. $req_P \leftarrow 0$

Process Q

1. $req_Q \leftarrow 1$
2. `wait(`$req_P = 0$`)`
3. `Critical Section`
4. $req_Q \leftarrow 0$

Initial state: $req_P = req_Q = 0$

Example: Mutual Exclusion Algorithm

Global variables: req_P and req_Q

Process P

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1.   reqP ← 1
2.   wait(reqQ = 0)
3.   Critical Section
4.   reqP ← 0
```

Process Q

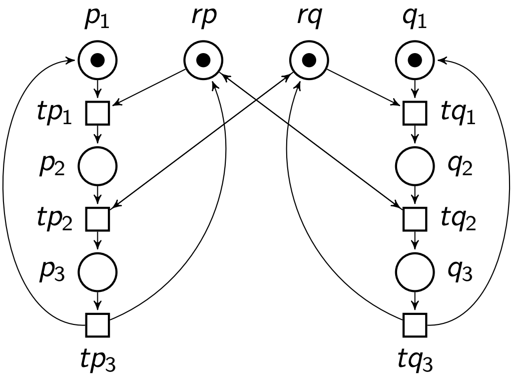
```
1.   reqQ ← 1
2.   wait(reqP = 0)
3.   Critical Section
4.   reqQ ← 0
```

Initial state: $req_P = req_Q = 0$

Properties to be checked:

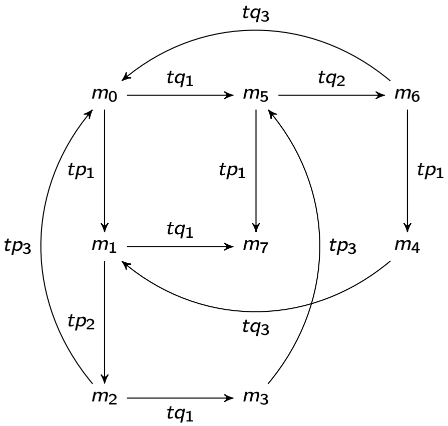
1. Mutual exclusion
2. Fairness
3. Order

Example: Mutual Exclusion Algorithm



Example: Mutual Exclusion Algorithm

$m_0 = p_1 + rp + rq + q_1$
$m_1 = p_2 + rq + q_1$
$m_2 = p_3 + rq + q_1$
$m_3 = p_3 + q_2$
$m_4 = p_3 + q_2$
$m_5 = p_1 + rp + q_2$
$m_6 = p_1 + rp + q_3$
$m_7 = p_2 + q_2$



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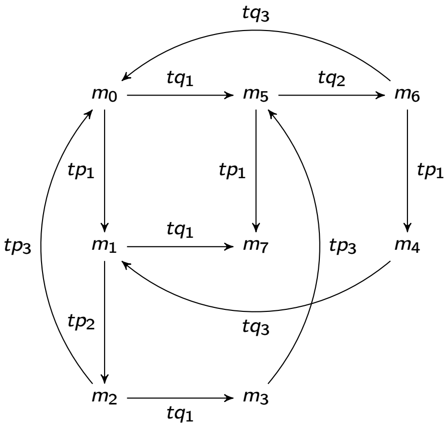

Compare with the previous reachability graph of the mutual exclusion example

Petri Nets modeling a hairdresser

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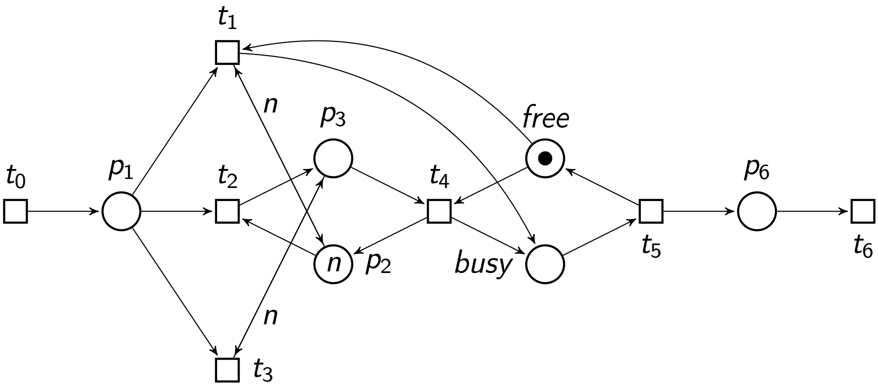
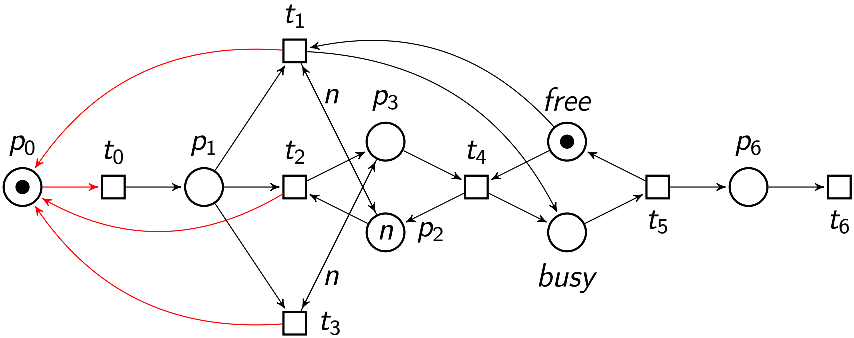


Petri Nets modeling a hairdresser (Cont.)



Petri Nets: Properties

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- N is structurally live iff $\forall m_0, (N, m_0)$ is live.

Relation Between Properties

- quasi-liveness VS Liveness ??

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Relation Between Properties

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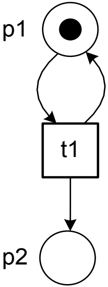
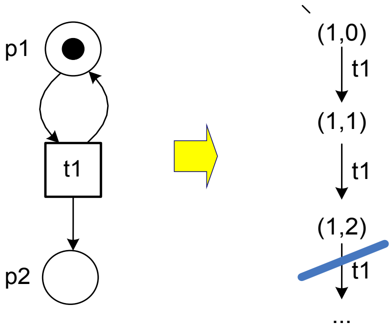
- liveness VS weak liveness ??
- m_0 home state and quasi live $\Rightarrow$ live ?? (if yes, proof)

Plan

Problem



Problem



- Notations:
  - new symbol $\omega \notin \mathbb{N}$ s.t.
    - $\omega + n = \omega$
    - $\omega - n = \omega$
    - $\omega > n$
    - $\omega \leq \omega$
  - $\mathbb{N}_\omega = \mathbb{N} \cup \{\omega\}$
  - For $q \in \mathbb{N}_\omega^m$, $q^{-1}(\omega) = \{p \in P \mid q(p) = \omega\}$

Definition (Coverability Tree)

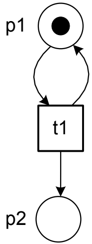
The coverability tree of a marked Petri net $\langle N, m_0 \rangle$ is a tree $\langle S, X \rangle$ where:

- nodes of S are labeled with vectors in $\mathbb{N}_\omega^m$ ($m =|| P ||$)
- edges of X are labeled with transitions in T.

Coverability Tree: Algorithm

1. Label the initial marking m_0 as the root and tag it "new".
2. While "new" markings exists, do the following:
   1. Select a new marking m and remove the "new" tag.
   2. If m is identical to a marking on the path from the root to m, then tag m "old" and go to another new marking.
   3. If no transitions are enabled at m, tag m "dead-end".
   4. While there exist enabled transitions at m, do the following for each enabled transition t at m:
      1. Obtain the marking m' that results from firing t at m.
      2. If, on the path from the root to m, there exists a marking $m'' \neq m'$ such that $m' \geq m''$, then replace $m'(p)$ ω for each p such that $m'(p) > m''(p)$.
      3. Introduce m' as a node, draw an arc with label t from m to m', and tag m' "new".
3. Output the tree

Coverability Tree: Example



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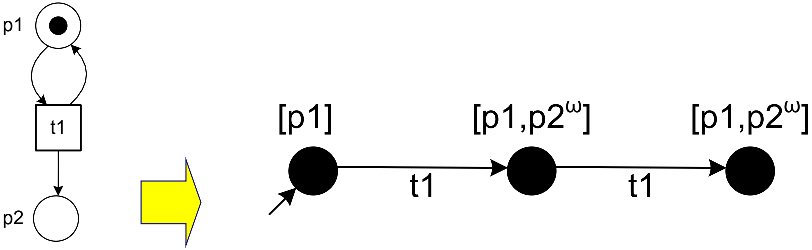
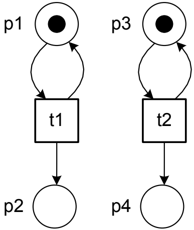


Coverability Tree: Another Example



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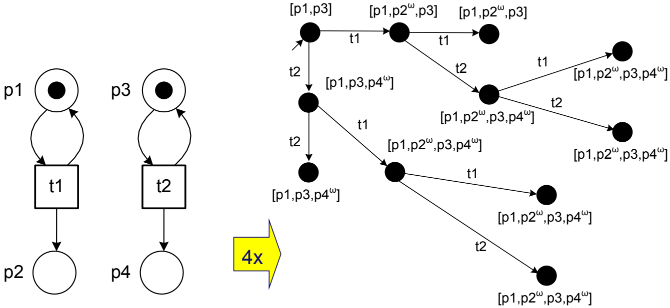

Coverability Graph

Take the coverability tree and merge nodes with identical labels

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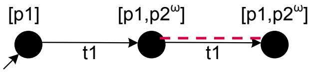

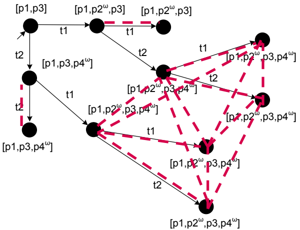

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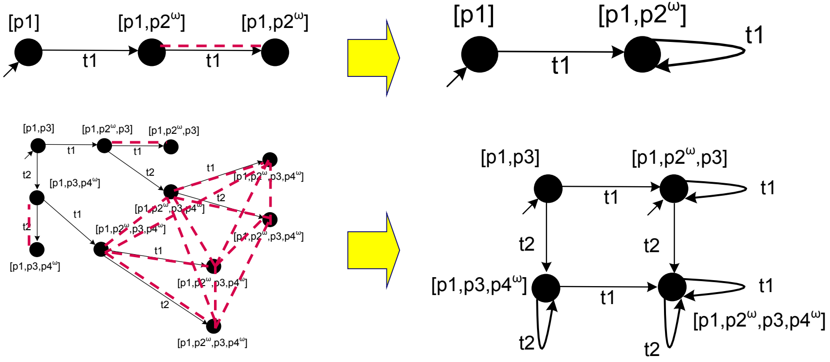

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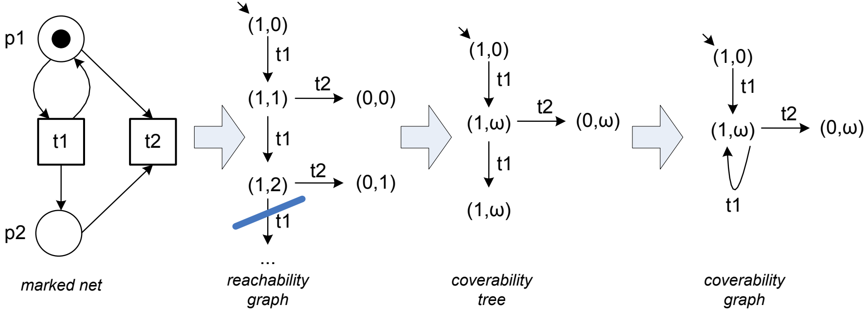

Properties

- The coverability tree/graph is always finite.
- The marked Petri net is bounded if and only if the corresponding coverability tree/graph contains only ω-free markings.
- The coverability tree/graph gives an over-approximation.
- Different Petri nets may have the same coverability tree/graph.
- Any firing sequence of the marked Petri net can be matched by a "walk" through the coverability graph.

Limitation: Loss of Information

The reverse is not true!!!!

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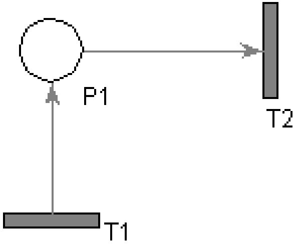

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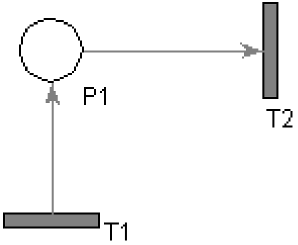

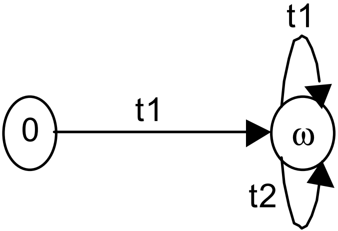

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Two nets with the same coverability graph!

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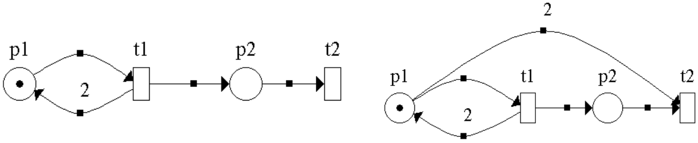

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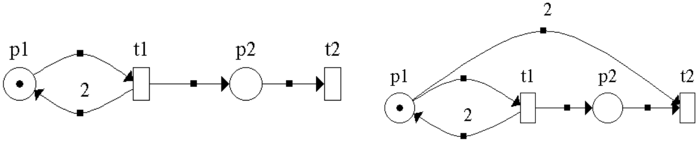
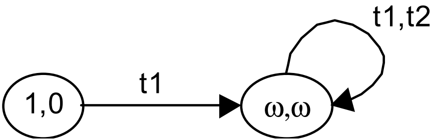
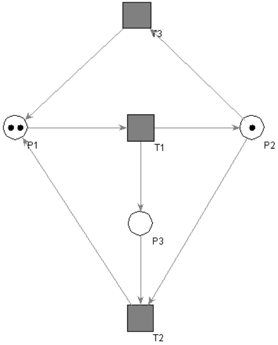




Coverability Graph: Exercice



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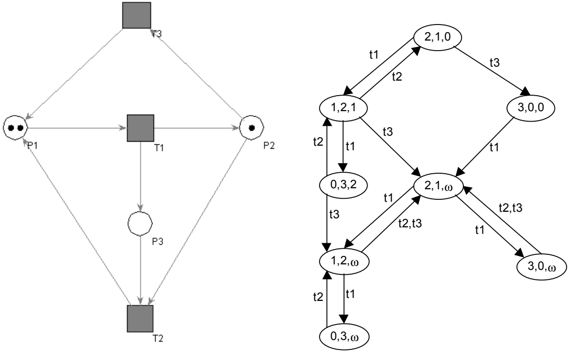
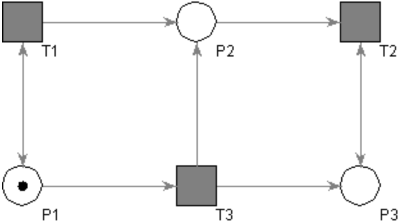


Coverability Graph: Another Exercice



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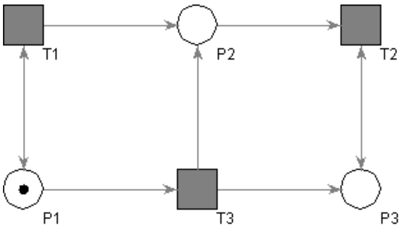

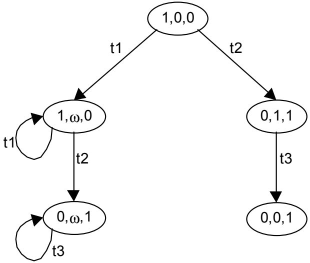

Plan

4. Formal Specifications
   - Petri nets
   - Coverability Graph
   - Linear Temporal Logic (LTL)

Temporal Logics

Two kinds of temporal operators

- sequence of expected events along one path
- e.g. **U**, **X**, **G**, **F**

Temporal Logics

Two kinds of temporal operators

- sequence of expected events along one path
- e.g. **U**, **X**, **G**, **F**

Insufficient: are all/some paths starting from a given state satsfy some property?

path quantifiers

- quantify paths starting from a state and satisfying a property
- e.g. **A**, **E**

Linear Temporal Logic (LTL)

Syntax

AP: a set of atomic propositions

$\varphi ::= \mathit{true} \mid$	logical constant true
$p \mid$	atomic proposition
$\neg\varphi \mid$	negation
$\varphi \wedge \varphi \mid$	and
$X\varphi \mid$	next time
$\varphi U \varphi$	Until

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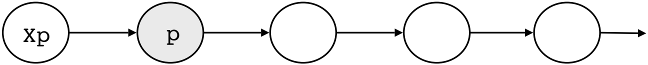
- $G\varphi$: now and always in the future
  - $G\varphi \equiv \neg F \neg\varphi$

LTL: Semantics

Express sequence of events along a path

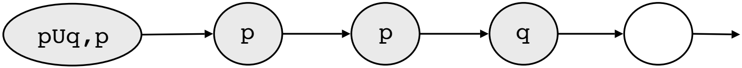
Operator **X** "next"



Temporal connectors

Express sequence of events along a path

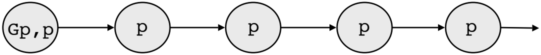
Operator **U** "p true until q true"



Temporal connectors

Express sequence of events along a path

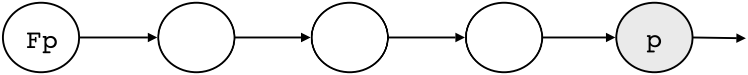
Operator **G** "always in the future"



Temporal connectors

Express sequence of events along a path

Operator **F** "eventually in the future"



LTL: Semantics

LTL is interpreted on infinite paths of a Kripke structure K.

$\pi = s_0 \longrightarrow s_1 \longrightarrow \ldots$

- $\pi \models p$ iff $p \in L(s_0)$
- $\pi \models \varphi_1 \wedge \varphi_2$ iff $\pi \models \varphi_1$ and $\pi \models \varphi_2$
- $\pi \models \neg\varphi$ iff not $\pi \models \varphi$
- $\pi \models X\varphi$ iff $\pi^1 \models \varphi$ (π^i = suffix of π starting at s_i)
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$$K \models \varphi \Leftrightarrow \forall \text{ path } \pi \text{ of } K,\ \pi \models \varphi$$

LTL: Exercice

Ecrire les formules LTL formalisant les propriétés suivantes :

1. One day, p will occur
2. p is always true
3. p occurs infinitely often
4. p and q are never true simultaneously
5. After an occurrence of p there will be at least one occurrence of q
6. If p_1 occurs infinitely often and p_2 occurs inifnitely often, then each occurrence of q_1 is followed by an occurrence of q_2.
7. Before each occurrence of p, there is at least one occurrence of q.
8. Between each couple of occurrence of p there is at least one occurrence of q.
9. No other coffee orders are accepted between the payment of the amount due and the removal of the cup.
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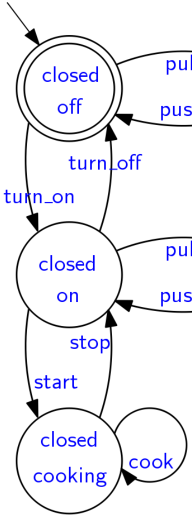
Does the property holds? counterexample?

$G(start \implies F\ stop)$



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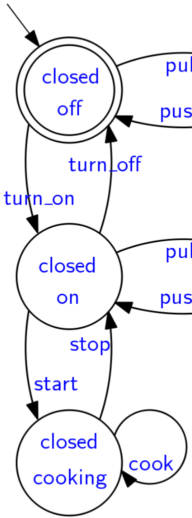
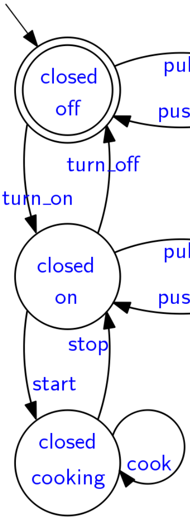


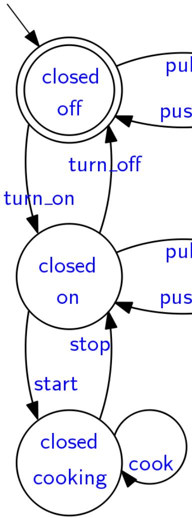
Does the property holds? counterexample?

G F turn_off



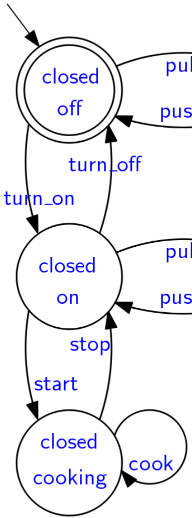
Does the property holds? counterexample?

G F turn_off ✓ (*pull push*)$^\omega$



Does the property holds? counterexample?

$G\ F\ (turn\_off \vee push)$



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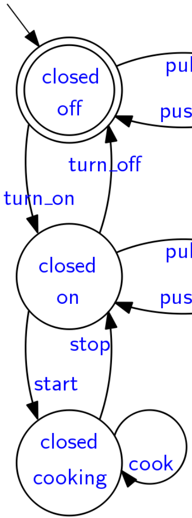
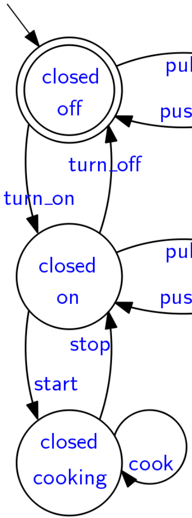


Does the property holds? counterexample?

$G\ False \vee F(turn\_off \vee push)$



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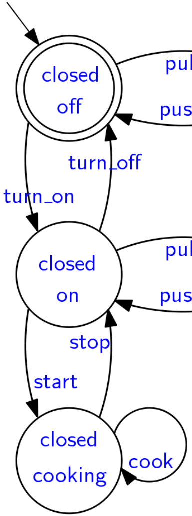
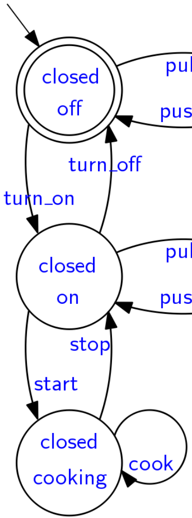


Does the property holds? counterexample?

$G(start \implies (cook\ U\ F\ turn\_off))$



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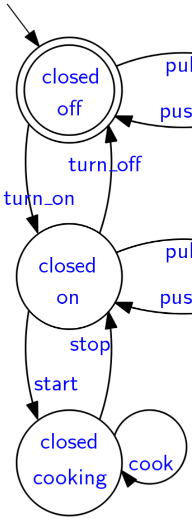

Outline

Plan

Büchi Automata

Definition

A Büchi automaton is 6-tuple $A = \langle \Sigma, Q, Q_0, F, \delta \rangle$ where:

- Σ is a finite alphabet
- Q is a finite set of state
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- F is a set of accepting states
- $\delta \subseteq Q \times 2^{\Sigma} \times Q$.

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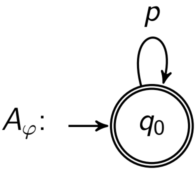
- Generalized Büchi Automata (State/Transition-Based)

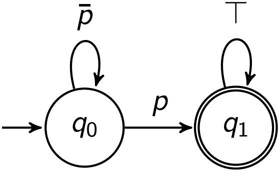
From LTL to Büchi automata

$\varphi = Gp$



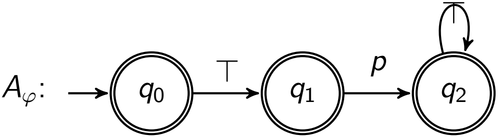
$\varphi = Fp$

A_φ:



From LTL to Büchi automata

$\varphi = Xp$



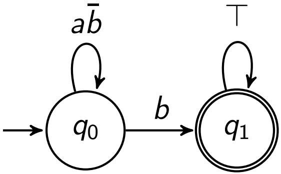
From LTL to Büchi automata

$\varphi = a \, \mathbf{U} \, b$

From LTL to Büchi automata

$\varphi = a \ \mathbf{U} \ b$

A_φ:



From LTL to Büchi automata

$$\varphi = G(p \implies F\, q)$$

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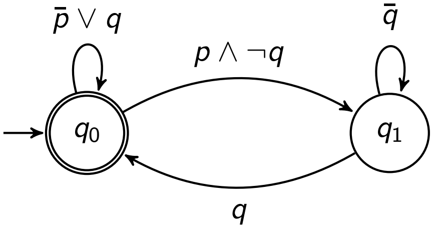

From LTL to Büchi automata

$\varphi = \neg G(p \implies F\ q)$

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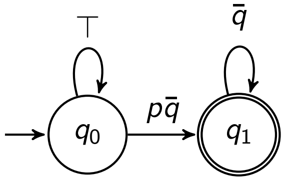
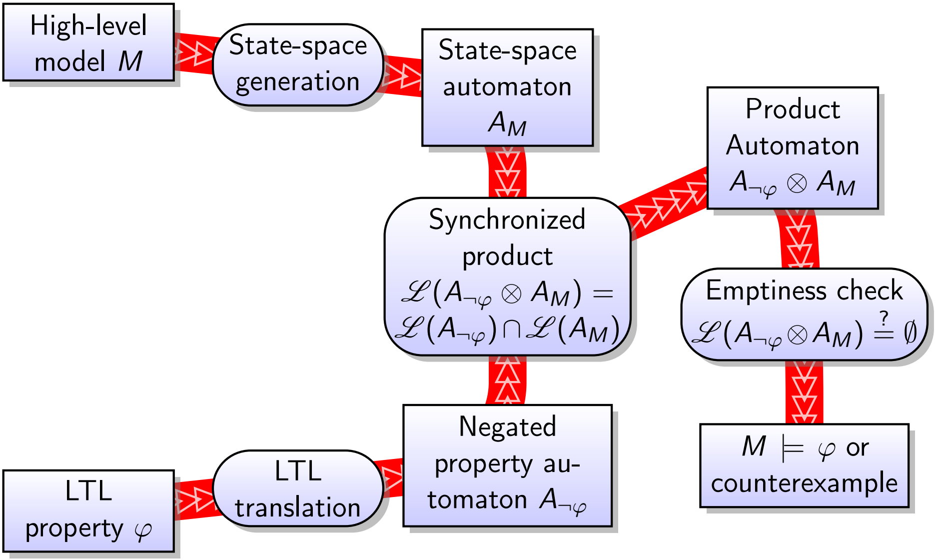


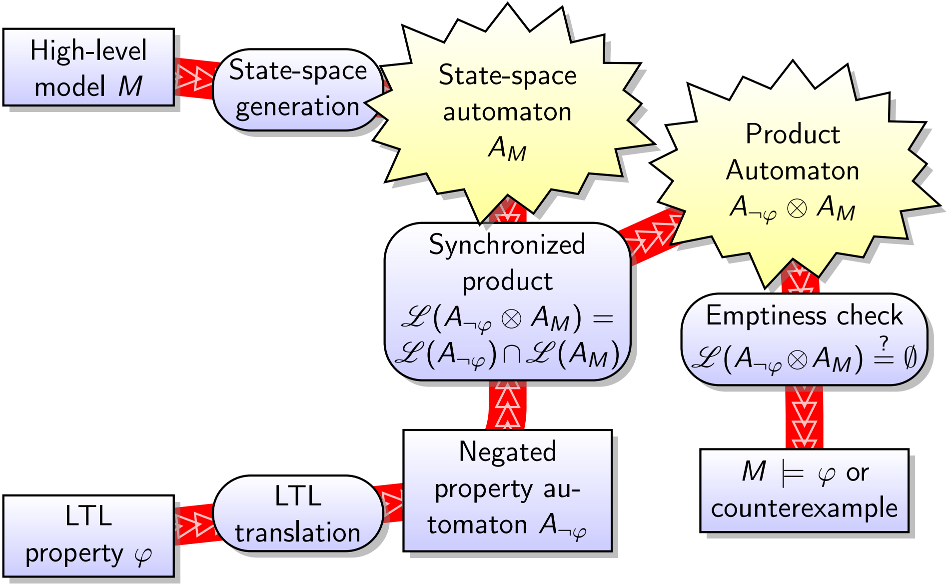
Plan

5. LTL Model Checking
   - Büchi Automata
   - Automata-Theoretic Explicit LTL Model Checking

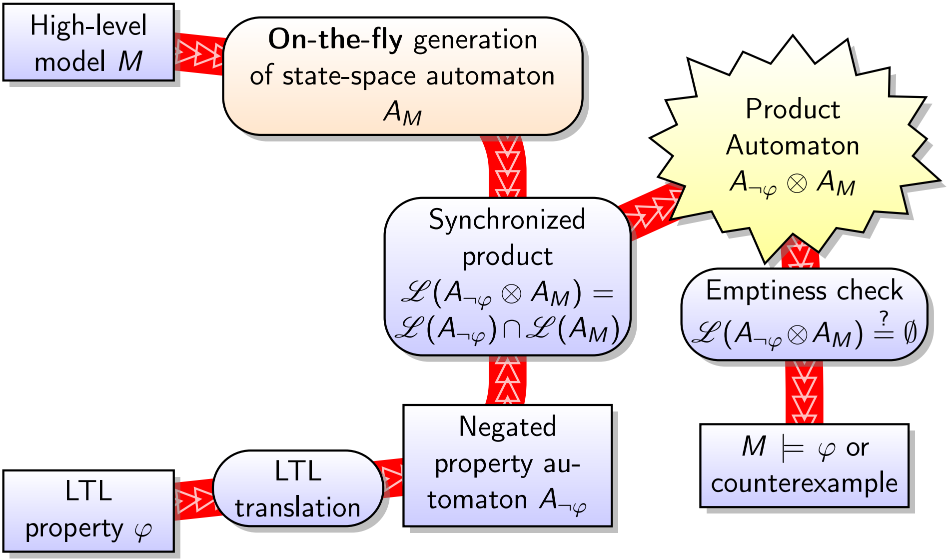
Automata-Theoretic Explicit LTL Model Checking



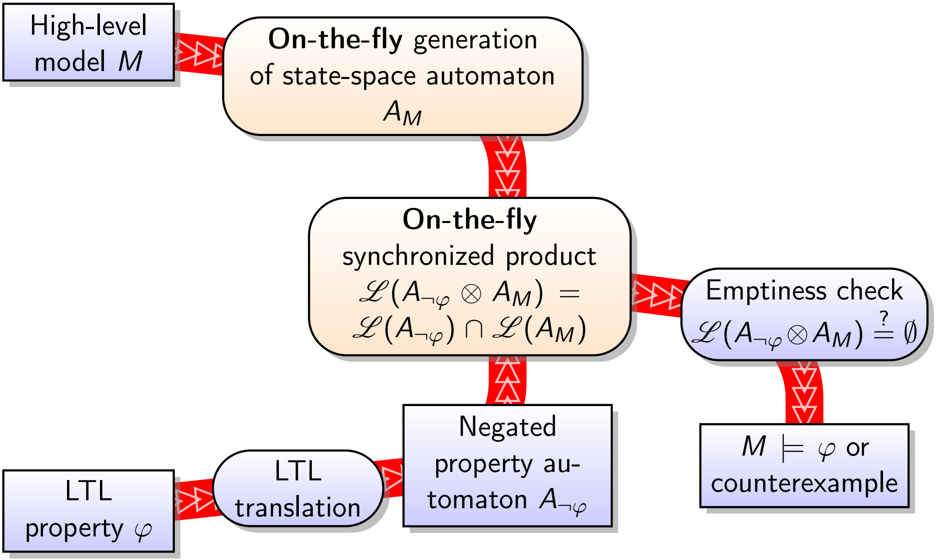
Automata-Theoretic Explicit LTL Model Checking



Automata-Theoretic Explicit LTL Model Checking

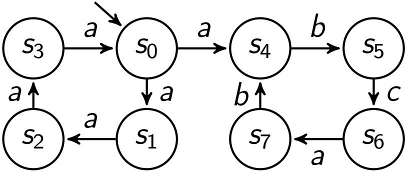


Automata-Theoretic Explicit LTL Model Checking

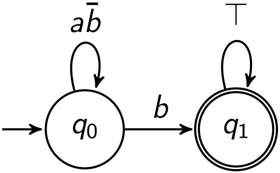


LTS × Büchi Automaton

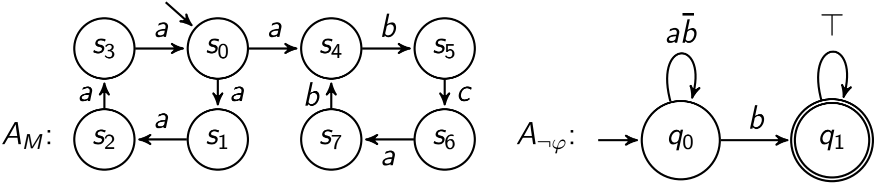
A_M:

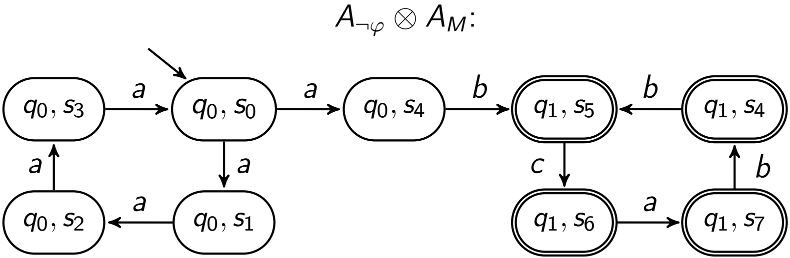


$A_{\neg\varphi}$:

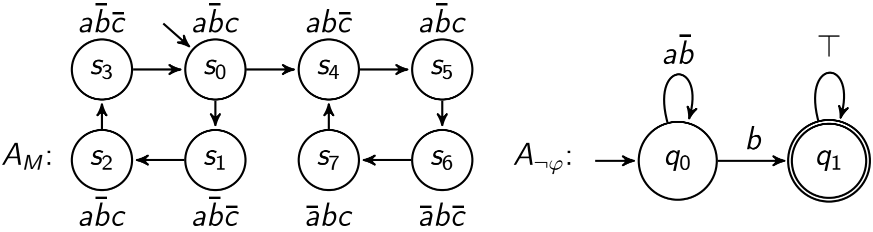


LTS × Büchi Automaton

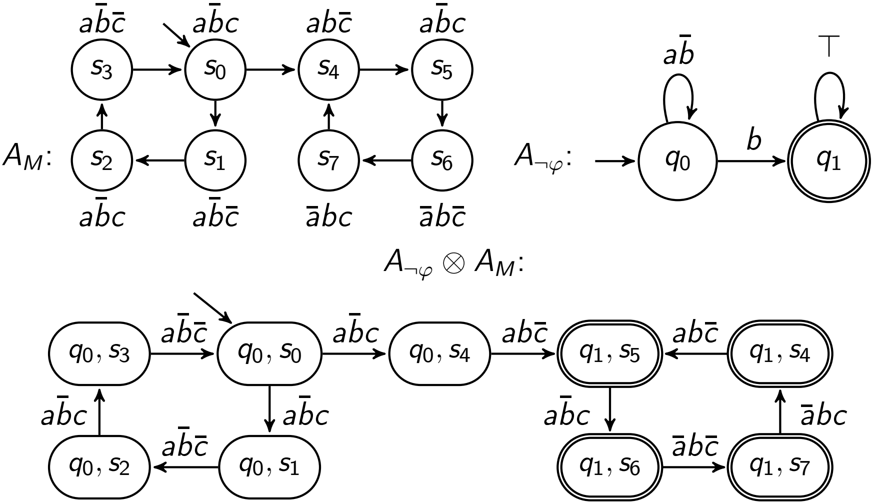




Kripke Srutcure × Büchi Automaton

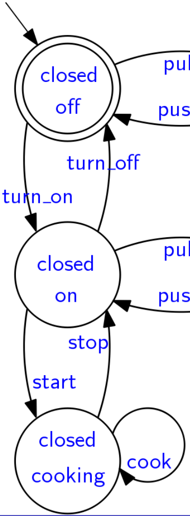


Kripke Srutcure × Büchi Automaton



LTS × Büchi Automaton: Exercice

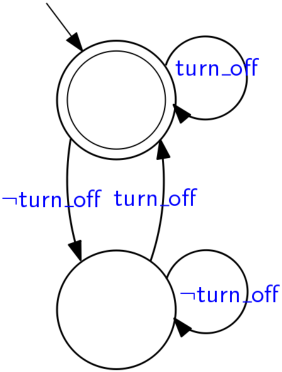
Let us demonstrate by model checking that *G F turn_off* is not satisfied



LTS × Büchi Automaton: Exercic

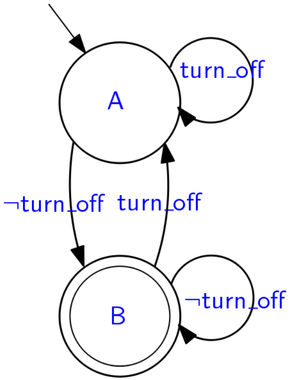
- Build a Büchi automaton with the same language as $\neg(G\ F\ turn\_off)$.
- Let us start from the unnegated formula: $G\ F\ turn\_off$

G F turn_off

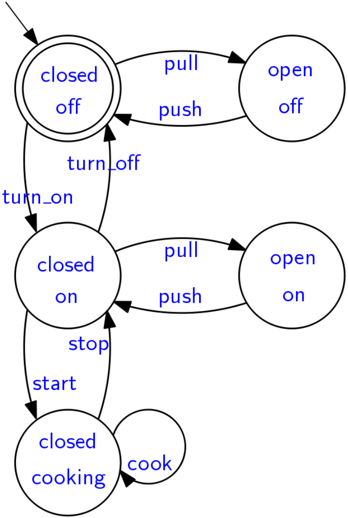


LTS × Büchi Automaton: Exercic

$\neg(G\ F\ turn\_off)$



LTS × Büchi Automaton



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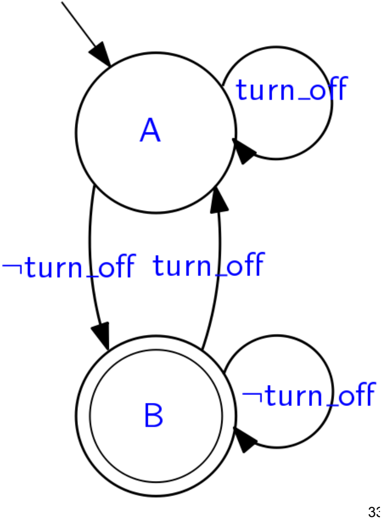
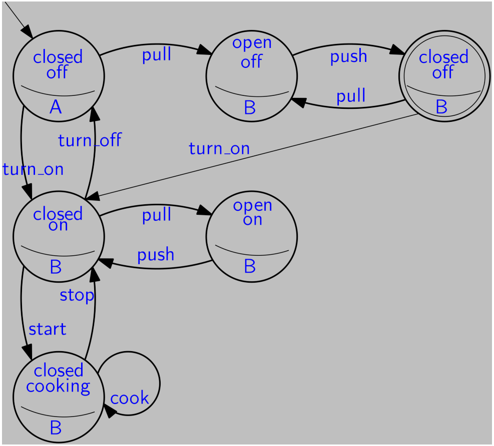


LTS × Büchi Automaton



Kripke Structure × Büchi Automaton: Exercice

Express (with LTL) and Check the three properties of the mutual exclusion Petri net model

$m_0 = p_1 + rp + rq + q_1$
$m_1 = p_2 + rq + q_1$
$m_2 = p_3 + rq + q_1$
$m_3 = p_3 + q_2$
$m_4 = p_3 + q_2$
$m_5 = p_1 + rp + q_2$
$m_6 = p_1 + rp + q_3$
$m_7 = p_2 + q_2$